SPECTRUM OF SMALL PERTURBATIONS IN PLANE CONCRETE FLOW

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The behavior of the eigenvalue spectrum in the general case of flow in a plane-parallel channel for small Reynolds number R was studied in paper [1]. The direct numerical calculation of the spectrum of small perturbations for plane Couette and Poisuille flow, carried out in [2-4], covers a comparatively wide range of variation of R, but was performed for fixed values of the wave number α in a fairly narrow range.

The present paper studies the eigenvalue spectrum as a function of the wave number over its whole range of variation for the case of plane Couette flow.

The problem reduces to finding the eigenvalues of the equation

$$\varphi^{\text{VI}} - 2\alpha^2\varphi'' + \alpha^4\varphi = i\alpha R\,(y - C)\,(\varphi'' - \alpha^2\varphi)$$
$$-1 \leqslant y \leqslant 1, \varphi'(\pm 1) = \varphi(\pm 1) = 0 \tag{1}$$

Here $\varphi(y)$ is the complex amplitude of the stream function of the perturbed motion, $C = X + iY$ is the required eigenvalue, the physical meaning of X is the phase velocity, while the case $Y < 0$ corresponds to a damping of the perturbation.

For small values of α the behavior of the eigenvalue spectrum can be described by the functions

$$X_k = 0, \quad Y_k = -\beta_k^2 / \alpha R, \quad k = 1, 2, 3, \ldots \tag{2}$$

Here β_k are numbered in increasing order of the modulus of the roots of the equation

$$(\beta \operatorname{tg} \beta + \alpha \operatorname{th} \alpha)(\alpha \operatorname{tg} \beta - \beta \operatorname{th} \alpha) = 0. \tag{3}$$

For large α the following asymptotic function is satisfied for all the spectral numbers

$$Y_k = -\alpha / R, \quad k = 1, 2, 3, \ldots, \tag{4}$$

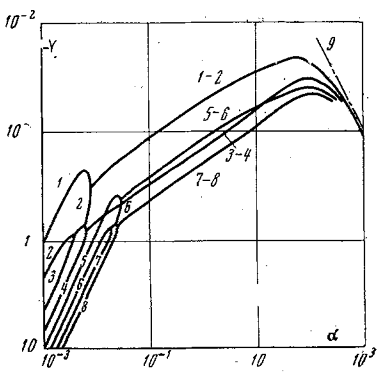

Fig. 1

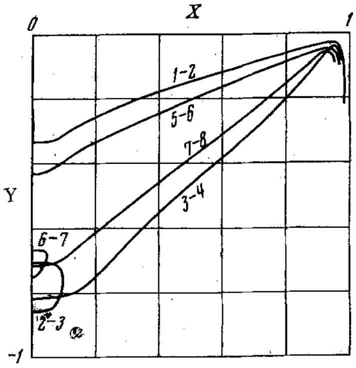

Fig. 2

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Thus the part played by direct numerical calculation is reduced to filling in the "gaps" between regions where the functions (2) and (4) are valid. The calculations were carried out on a BESM-6 digital computer using a method of stepwise integration [5], modified for asymmetric velocity profiles.

The functions $Y_k(\alpha)$ are given in Fig. 1 for $R = 10^4$, $k = 1, \ldots, 8$. Line 9 corresponds to function (4). The disposition of the eigenvalues in the complex plane $C = X + iY$ is shown in Fig. 2 for variable α. The line $X = 0$ is an axis of symmetry.

In accordance with general laws [1] the perturbations are damped for small α, remaining monotonic ($X = 0$). The eigenvalues exhibit multiplicity points for certain values of α, after which the perturbations begin to have a wave-like character, while a pair of waves arises similar in damping Y and phase velocity $|X|$, but travelling in opposite directions. Each of these waves propagates in its own half of the stream since the eigenfunction is usually nonzero only in a small neighborhood of the point $y_c = X$.

However, the spectral structure is more complicated here than in the cases considered in papers [2-4], and new interesting effects appear, associated with the multiplicity points. After the multiplicity point of the second and third eigenvalues for $\alpha = 0.0038$, $|X|$ at first increases for the wave pair as α increases, but subsequently decreases again to zero for $\alpha = 0.0057$, where the second and third eigenvalues again have a multiplicity point, subsequently separate and become purely imaginary. As α increases still further Y_3 decreases until the third eigenvalue has a multiplicity point with the fourth (which is purely imaginary up to this point) for $\alpha = 0.006$, and subsequently forms a complex conjugate pair of decrements iC_k ($k = 3, 4$). Somewhat later for $\alpha = 0.008$ there is a confluence of the second and first eigenvalues, after which a complex conjugate pair is formed iC_k ($k = 1, 2$).

This set of four eigenvalues does not suffer any further interchanges. For all k, $|X|_k$ increase and tend to unity for $\alpha \to \infty$, i.e., the perturbations are localized at the walls. The quantities $Y_{1,2}$ and $Y_{3,4}$ also increase, attain their maximum values 0.021 and 0.033 for α equal to 59 and 105 respectively. They then decrease and end up as the asymptotic function (4).

The second set of four eigenvalues $k = 5, 6, 7, 8$ behaves in a similar manner. It is characteristic that if we observe the intersection of the functions $Y_k(\alpha)$ for various k and intermediate values of α, the decrements once again rearrange themselves in the same order as for small α when we approach the asymptotic function (4). It is clear from Fig. 2 that the points where Y reaches its greatest value for all spectral numbers k, lie in the neighborhood of $C = \pm 1$. Calculations show that the points $C = \pm 1$ are limiting points for the eigenvalues when α and k are arbitrary fixed values and $R \to \infty$.

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